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Investigation of Iterative Algorithms For Evaluation of Capital Structure and Cost

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Determination of structure and correct calculation of a company's capital value is an essential; theoretical and practical problem for corporate finance. The proportion between the company's equity and borrowed capital determines the risk and profitability of the company and, consequently, the welfare of its owners. The most common recommendation is to evaluate the stricture of capital based on market proportions between indebtedness and equity. However, market proportions most often deviate from values obtained through analytical calculations. This means that weak efficiency of the market brings about inconsistency between the input data and the results, which are calculated from them. Second, not all companies have a representative market quotation. There is a question, then: how can we correctly evaluate capital and its market structure for individual projects and companies in general? The work presented below is dedicated to the iterative method for evaluation of fair structure of capital as suggested in (Limitovsky M.A., Minasyan V.B. 2010), and to the proving of consistency of this method for a very large number of companies.

Keywords: company's value, structure of capital, free cash flow, iterative method, principle of contracting mappings, fixed point of mapping, duration of cash flows, convergence of recurrent process.

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Investigation of iterative algorithms for evaluation of capital structure and cost

1. Relation between the company's value and the structure of its capital

By definition, structure of capital is essentially share of each type of capital in the total capital of a company or an investment project. In particular, the most well-known method for evaluating a company or a project is the WACC (weighted average cost of capital) technique, in which, to evaluate a company or a project, free cash flows (FCF) are discounted by the moment of evaluation using the WACC value as the discount rate:

$$V = \frac{FCF_1}{1+WACC} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + \cdots + \frac{FCF_n}{(1+WACC)^n} \qquad (1)$$

where n = projected period;

FCFo, FCF_1, FCF_2, … FCFn = free cash flows or flows from assets as calculated for the projected period. The last flow includes the present value of all future cash flows of the post-projected period;

V = value of the company's assets. To evaluate equity of the company (i.e. shareholders' equity), we must deduce the borrowed capital from this value:

$$V = D + E, \text{ or } E = V - D \qquad (2)$$

WACC is found from the well-known formula:

$$\text{WACC} = k_d(1-T)\cdot w_d + k_e \cdot w_e, \qquad (3)$$

- Where: k_d = value of the borrowed capital (average);
- w_d = share of the debt in the corporate capital structure;
- T = profit tax rate;
- k_e = average cost of the corporate equity;
- w_e = equity share in the corporate capital structure.

It is clear that the shares w_e and w_d, which essentially characterize the capital structure, determine the WACC, the company's value (V), and evaluation of its equity (E).

Along with influence on the WACC in general, capital structure also affects the cost of equity. Cost of equity k_e, i.e. rate of return, which share investors take into account, depends on their risk. And the finance risk, in turn, is determined by the ratio between the creditors' capital and the owners' capital.

Formally this can be presented as follows: If we assume the well-known Capital Asset Pricing Model (CAPM) as a basis for determining equity value, then k_e will be found from the following formula:

$$k_e = R_f + \beta \cdot (R_m - R_f) \text{, or} \tag{4}$$

$$k_e = R_f + \beta \cdot \Delta R \text{, where}$$

R_f = risk-free investment rate (rate of return on state discount securities in stable economies), % per annum;

R_m = average returns on the market portfolio (average annual growth of the market index, such as S&P500 in the USA, FTSE in England etc.), % per annum;

$\Delta R = R_m - R_f$ = market premium for risk of investment into shares, % per annum;

β = indicator of systematic risk on shares of a specific company. It is calculated in a centralized manner by such agencies and institutions as Barra International, Meryll Lynch etc. It is determined as a coefficient of regression in the equation of connection between returns on a specific share and the market in general (the market index).

If we use the famous formula by Robert Hamada (Hamada R. 1972), coefficient β may be presented as a product of two coefficients:

$$\beta = \beta_0 \times \beta_1 \text{, where:}$$

β_0 = "unlevered" coefficient reflecting the degree of business risk of a corporation;

β_1 = corrective coefficient reflecting the extent of the financial risk, because a company, which uses borrowed funds, creates an additional risk for its shareholders. According to the well-known formula:

$$\beta_1 = 1 + D/E\ (1 - T) \text{, where} \tag{5}$$

D/E = ratio between borrowed funds and the equity (the financial leverage);

T = profit tax rate (fraction of a unit). In particular, in famous papers, such as (Damodaran A., 2004; Peterson D., Peterson P. 1996), this is the technique recommended for adjustment of systematic risk.

The problem is that it would be incorrect to use Hamada's equation for real-world conditions, since the equation is the direct consequence from the second Modigliani-Miller law including the taxation and the introduction, as a mandatory condition, of a non-risk nature of the corporate debt (i.e. the debt granted and received at a risk-free rate). In conditions where debt is not of a non-risk nature, use of the Hamada equation may bring about errors and incorrect idea of changes in the value of the company depending on changes in leverage. Authors are more correct (in particular, (Lumby S., Jones C. 2004)) when they use a different relation between coefficient ß and financial leverage, namely:

$$\beta_0 = W_e \cdot \beta_e + W_d \cdot \beta_d, \qquad (6)$$

where:

$$W_e = \frac{E}{D(1-T)+E}; \qquad (7)$$

$$W_d = \frac{D(1-T)}{D(1-T)+E}; \qquad (8)$$

$\beta_d, \beta_0, \beta_e$ = systematic risk of corporate debt, the company's assets, and its equity.

Use of these equations, as in the case with the Hamada equation, on the one hand, is based on ignoring the transaction/agency expenditures and bankruptcy expenditures. On the other hand, such equations are much more adequate than reality, since they assume that the creditor did not take the risk-free position, and the debt involves its systematic risk. When such equations are used, as the creditor assumes part of the risk, the owner's risk decreases accordingly, whereas the weighted average capital cost does not change as a result of the re-distribution of risk between the creditor and the owner, and instead remains a constant value, independent from the specific percentage rate:

$$\text{WACC} = k_0 \cdot (1 - w_d \cdot T) \qquad (9),$$

Where $k_0 = R_f + \beta_0 \cdot \Delta R$

Thus, capital structure management is an integral part of the company value based management. It determines cost of the company's equity, weighted average capital cost, and, finally, value of the company.

2. Single-step methods for calculating a company's capital structure and cost

In practice, in simplified calculations for determining capital structure and cost, shares of each type of capital are used, which are expressed as follows:

- In balance valuation;
- As shares of capital invested in a project or a company;
- In market valuation.

However, all these methods are theoretically not quite correct, whereas in practice they can bring about errors and distortions in valuation analyses.

1). The first solution, which is attractive as a result of its simplicity, consists in using the appropriate *balance proportions* between the debt and the equity. Further, because structure of the capital changes every year (or even every month), both capital structure and its average weighted value should be changed. For example, as a debt is repaid, its share in the total capital decreases. Therefore, it would be logical to discount free cash flows at changing discount rates, which is actually suggested by some authors (Guidelines. 2000; Holden C.W., 2004).

Why, strictly speaking, may we not use the balance structure of capital in evaluation analyses? Any evaluation is fair solely as of the date of such evaluation, and only in connection with objectives, for which such evaluation is done. This means that non-current (mostly historical) balance data do not reflect the current situation, because such data, at best, were correct as of the date of the respective transaction. We say "at best", because not all of the assets are reflected in the balance sheet, and, accordingly, not all of the capital is taken into account in determination of its structure. Market prospects, any non-trivial commercial idea or an access to limited resources are actually the most valuable assets, which are present within the project. However, such assets are not placed on the balance; still, they are valued by the market. Presence – or, rather, absence – of unaccounted assets in the balance sheet considerably distorts the computation results. When we make a calculation based on balance sheet data, we can obtain huge financial leverage, with the borrowed capital exceeding the equity several times (or even dozens of times). If, in addition to it, we use the Hamada equation to adjust the coefficient ß, we can get a huge (and totally unrealistic) cost of equity. All this means that we should avoid using balance sheet data in calculating the capital structure.

A reasonable and correctly evaluated cost of capital should reflect the idea of the company as of the moment of valuation – but not some time ago, when the balance calculations were made. When we evaluate new projects, we should base on the cost of new capital, i.e. the cost of capital, which should be covered by the returns of the current project in future – but not on the rates, at which the capital was obtained by the company in the past.

2). In some cases, calculations of the capital structure for a project are *based on shares of capital invested in the project* (Limitovsky M.A. 2004). Here, it should be taken into account that earnings received from a project and capitalized at its future stages should be viewed as equity investments. And, again we may encounter the problem of change of the financial leverage. The matter is that a company's equity as of the date of involvement in project with a positive net present value (NPV) becomes increased by the size of the NPV. It happens exactly at the moment when the company makes the decision to launch the project or to take part in it. Thus, the company's equity at the moment must be increased by the NPV, and the structure of the capital for the project will not correspond to the structure of material investments in the project. The calculation algorithm will feature an inconsistency between the initial data and the calculation results.

The principal methodological difficulty of evaluation in accordance with the WACC technique consists in the fact that one should know WACC to determine NPV, whereas for calculation of WACC, NPV should already be present within the structure of the capital (Refer to the example in (Limitovsky M.A., Minasyan V.B. 2010)).

3). In basic manuals on corporate finance, the most common recommendation is to ***use market valuations of equity and borrowed capital*** in WACC calculations. Market valuation of equity is essentially capitalization of the company's shares; whereas market valuation of the borrowed capital is essentially capitalization of its bonds.

However, there are three "contras" against this technique. First, not all of the companies, which need valuation, quote their shares and bonds (here we mean *only* shares and bonds). Second, the real market may reflect the value of assets not quite adequately because of the non-representative nature of quotations of an individual issuer and/or non-efficiency of the stock market itself. Third, this method contains an intrinsic contradiction. In fact, the main purpose of evaluating a company, which is quoted at the market, is to find out underestimated assets. Consequently, market valuation is deemed imperfect and not quite correct; and the valuator, supposedly, provides his, more correct, valuation. However, to reach such valuation, he should base his calculations on "incorrect" market proportions. As a result of such inconsistency, what we have is a lack of matching between the input data and the calculation results. (Refer to the example in (Limitovsky M.A., Minasyan V.B., 2010])

A correctly valuated capital structure should not feature such inconsistencies, and its valuation should be based on conditions of a market without price-related irrationalities.

Many authors believe that ***capital structure should be purpose-oriented rather than factual***. A purpose-oriented (i.e. reasonable and conforming to the credit rating) structure of capital must be used for the following reasons: Assume that at a certain period of time the company formed an optimum (for itself) structure of the capital. When the company repays an old debt, its share of borrowed capital is reduced, and it acquires an opportunity to renew borrowings. Therefore, in future it will be able to reproduce a capital structure, which is optimum for itself. If the management of the company fails to do so, this will be their problem, which should not affect the valuation results, provided that the opportunity to build up the debt does exist. Speaking again of projects – if we did not take into account that the project created new assets, and such assets allowed creating new debts up to the optimum level of leverage, it would mean that we were underestimating the role of such projects. Instead, we would be funding new projects and overestimating them. All this testifies to the fact that if the company's credit rating does not change as a result of implementation of a project, the structure of its capital should be deemed constant.

However, postulating that structure of capital should be purpose-oriented is not enough. If we take it "off the mark", without basing on calculations of the factual structure of the capital, it would mean that such "purpose-oriented" structure is unreasonably arbitrary. Therefore, before deciding whether the existing structure of the company's capital is an optimal and purpose-oriented one, the factual structure of such capital shall be calculated correctly.

Thus, each of the above-listed single-step methods for determining capital structure has several drawbacks. They create a distorted impression of the real capital structure of a company or a project, and are inconsistent and not quite correct.

In respect of a single project, it may be said that to evaluate the NPV one should know the weighted average cost of capital (WACC). However, to obtain such value, one should know the ratio between equity and borrowed capital. And the equity includes the NPV, i.e. the final result of the calculations.

In respect of the company as a whole, it may be said that value of a company (V) is essentially a sum of borrowed capital and equity (2), and is calculated by discounting its cash flows at the WACC rate (1). This rate is determined based on the capital structure; to evaluate it, one should know the ratio between equity and borrowed capital. This means that, with known amount of borrowed capital, one should also know V.

3. Iterative (multi-step) algorithms for evaluating capital structure and cost as suggested by other authors

Thus, the algorithm features a cyclic nature: the final valuation result cannot be obtained without knowing the capital structure, whereas, to determine the capital structure, the final valuation result should be known. We may conclude that calculation of capital structure for a company, which has no market quotation, should be done in several iterations in accordance with a multi-step algorithm. Literature provides us with two most famous algorithms of the type, the Evans-Bishop method and the Pratt-Martin method. Both of them relate to valuation of companies generating cash flows.

The algorithm suggested by Frank Evans and David Bishop (Evans F. Ch., Bishop D.M. 2004) can be briefly described as follows: At stage 1, to calculated WACC, debt/equity ratio as per balance sheet is used. It is assumed that the carrying value of the debt corresponds to its market value (which assumption, in many cases, for closed companies, is true, because borrowed capital for such companies is available at a market interest rate, which is affordable for the company. However, it is not always true.

Then the calculation of value of the invested capital is carried out by the discounted cash flow (DCF) method (or by the capitalization method), and the value of the debt (D) is deduced from the resulting value .The obtained equity market value is then compared to the debt value (i.e. D/E ratio is found). This ratio usually differs from the initial ratio (D/E), which was assumed for WACC calculation. The new ratio (D/E) is used for the new WACC calculation and for re-calculating the market value of the invested capital of the company. Such re-calculation is done until the resulting ratio between the debt and the market value of the company's equity (the D/E value) is stabilized and becomes equal to the D/E value assumed for WACC calculation.

The weakness of the Evans-Bishop algorithm consists in the fact that for iterative calculations the cost of equity (k_e) is assumed to be constant. In our opinion, this is not correct, because iterations each time change the structure of the company's capital. It is known that growth of the share of debt in the capital structure increases the risk for shareholders and, accordingly, the cost of equity. This relationship is expressed, for example, by the above-specified equations (6, 7, and 8). The algorithm is heuristic in the sense that the authors do not prove that it always has a solution (and, furthermore, only one solution).

The algorithm described by Pratt and Martin (Pratt Sh., 2006) includes the following steps:

- Balance-sheet valuations of equity and borrowed capital are introduced, and the balance structure of the capital and the financial leverage are calculated. Also, unlevered coefficient ß and the cost of borrowed capital are provided;
- From the Hamada equation, (5) levered ß is calculated using the financial leverage found at the first step;
- Equity cost is calculated using the CAPM;
- From the already known cost of borrowed capital and cost of equity, WACC is calculated;
- Using the Gordon formula (the capitalization method), the company is evaluated taking into account the obtained WACC:

$$V = \frac{FCF_1}{WACC - g};$$

FCF_1 = expected free cash flow of the subsequent period;

g = mean average rate of long-term growth of the cash flow.

From this figure, value of borrowed capital is deduced. Equity figure E is obtained, which is compared to the equity, which was assumed at the first step of the calculation. If these figures are equal, the calculation is complete. If the figures are not equal, the initial equity valuation is replaced by the calculation result, and the calculation is repeated until equal figures are obtained.

However, the algorithm has several drawbacks. First, the authors fail to prove that it always has a solution (and, furthermore, only one solution). Second, to adjust the systematic risk coefficient, the Hamada equation is used, which, as we have already mentioned, is not correct for companies, where the creditor's risk is different from zero. However, in such companies the debt, by definition, cannot be risk-free, and, consequently, usage of the Hamada equation is an unreasonable simplification. Third, the authors of the algorithm suggest using the capitalization method to assess the business in cycle. To use this method, the company should be stable and generate infinitely growing cash flows with a constant rate of growth, which is a very rare case in reality. At each step of the Pratt and Martin algorithm, the structure of the company's capital is changed; however, the cash flow growth rate, as initially selected, does not grow – thought it is possible that the credit rating of the company does change. Furthermore, by using the Gordon formula, we separate the task of determining the capital cost from the next task of valuation of the company; and the valuation results as obtained at the next step using the DCF method become inconsistent with the structure of the capital, which was calculated in accordance with the above-presented algorithm using the capitalization method.

4. Algorithm for calculation of capital structure of a company generating cash flows

As can be seen from the above, iterative algorithms for calculation of structure and cost of the capital eliminate the inconsistencies between the initial data and the calculation results. Furthermore, such algorithms are more reasonable in terms of theory. However, the approaches described above contain considerable methodological drawbacks. Such approaches are suitable not for all possible types of value generators; and it has not been proven that they have a single solution.

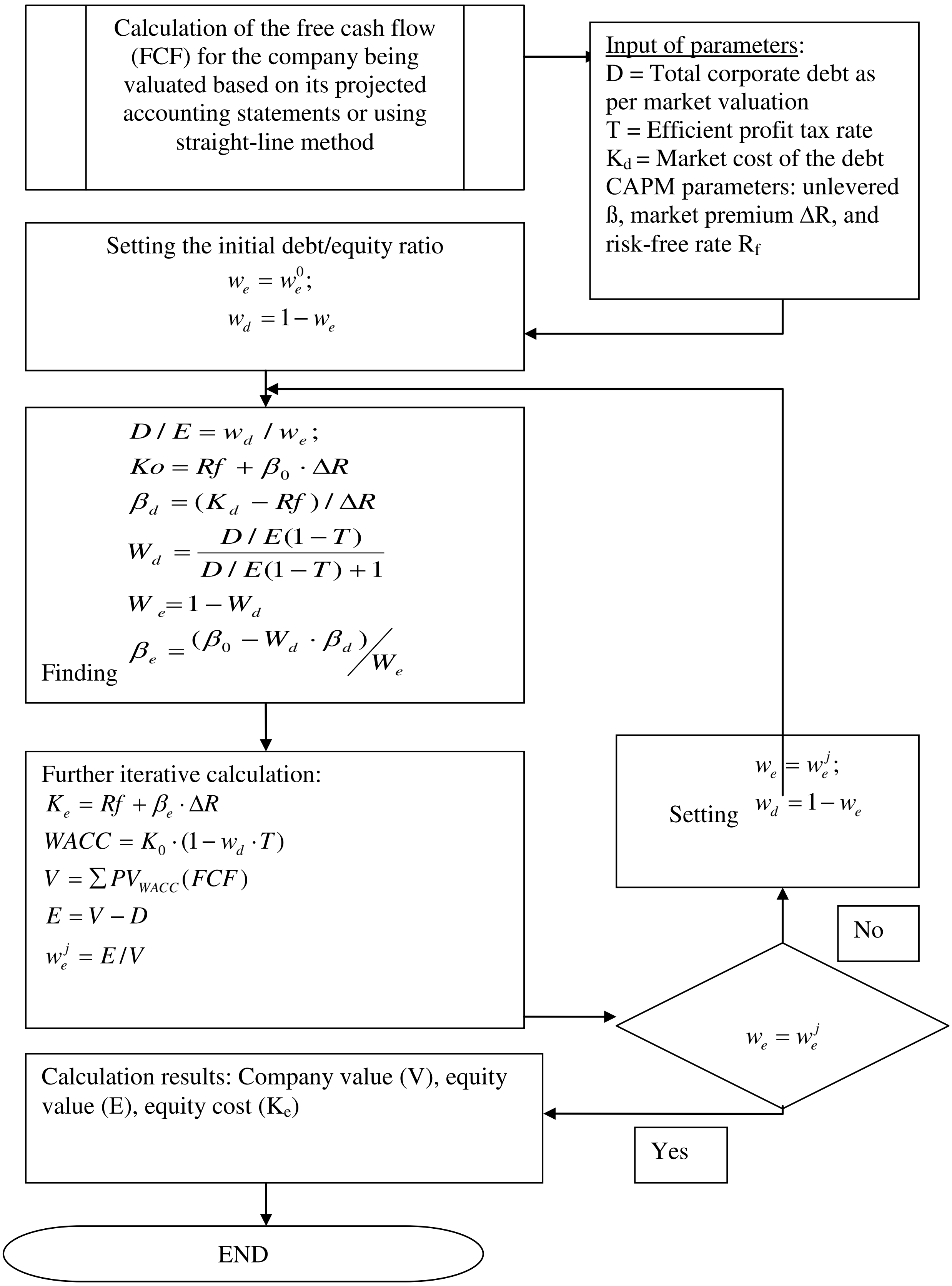
In the work by (Limitovsky M.A., Minasyan V.B. 2010) an algorithm for evaluating a company generating cash flows was suggested as shown in Figure 1.

As opposed to the closest (in its principle) Pratt-Martin algorithm, this technique:

- Does not use the Hamada equation, i.e. does not assume that the creditor is 100% protected and grants the borrowed capital to the company at a risk-free rate. On the contrary, our algorithm utilizes the assumption that the borrowed capital has its own systematic risk, i.e. β coefficient, which is different from zero;
- Uses rather the DCF method than the capitalization method for cyclic evaluation of the business; and the task of evaluating the capital structure is not separated from the subsequent task of business valuation;
- Offers a calculation, which is in no way connected to balance proportions in the capital structure;
- Suggests a slightly different condition for termination of the cycle: In our algorithm this is the criterion of equality of the initial capital structure and the capital structure obtained as a result of the calculation, whereas in Pratt's algorithm it is the equality of the appropriate value of equity. This last condition is not essential; however, the capital structure verification slightly simplifies the algorithm by making it more

logical. Besides, it allows introducing rather market value and cost of debt than the carrying value in the algorithm.

Fig. 1: Algorithm for valuation of a company generating cash flows


Calculation of the free cash flow (FCF) for the company being valued based on its projected accounting statements or using straight-line method
Input of parameters:
D = Total corporate debt as per market valuation
T = Efficient profit tax rate
K_d = Market cost of the debt
CAPM parameters: unlevered ß, market premium ΔR, and risk-free rate R_f
Setting the initial debt/equity ratio
$w_e = w_e^0;$
$w_d = 1 - w_e$
$D/E = w_d / w_e;$
$Ko = Rf + \beta_0 \cdot \Delta R$
$\beta_d = (K_d - Rf)/\Delta R$
$W_d = \frac{D/E(1-T)}{D/E(1-T)+1}$
$W_e = 1 - W_d$
Finding $\beta_e = (\beta_0 - W_d \cdot \beta_d) / W_e$
Further iterative calculation:
$K_e = Rf + \beta_e \cdot \Delta R$
$WACC = K_0 \cdot (1 - w_d \cdot T)$
$V = \sum PV_{WACC}(FCF)$
$E = V - D$
$w_e^j = E/V$
Setting $w_e = w_e^j;$ $w_d = 1 - w_e$
No
$w_e = w_e^j$
Yes
Calculation results: Company value (V), equity value (E), equity cost (K_e)
END

5. Sufficient condition for existence and uniqueness of a reasonable capital structure for a company or a project

In this section, we will prove the existence and uniqueness of a reasonable capital structure using the example of a company generating cash flows. In the beginning, as an example, we will take the case when the expected cash flows of the company are positive.

Using m (m=1, 2 …), we will mark the number of each subsequent step in the iterative process for valuation of the market structure of capital and cost of capital as specified above. Accordingly, to all figures as calculated at the m^{th} step we will assign the index *m*. In particular, as $w_e^{(m)}$ and $w_d^{(m)}$ we will mark, accordingly, the share of equity and debt in the corporate capital structure as calculated at the m^{th} step of iterations; as V_m we will mark the value of the company's assets, and as $WACC_m$ we will mark the weighed average capital cost as calculated at the m^{th} step of the iterative process.

It is clear that

$$w_d^{(m)} = 1 - w_e^{(m)}.$$

Hence, the assumption of convergence of $w_e^{(m)}$ values with the growth of number of iterations, i.e.:

$w_e^{(m)} \to w_e^o = const$, equivalent to convergence of $w_d^{(m)}$ values with the growth of number of iterations, i.e.: $w_d^{(m)} \to w_d^0 = const$

Our objective is to prove the convergence.

It is clear that:

$$w_d^{(m)} = \frac{D}{V_m}.$$

Using equation (1), we find that:

$$w_d^{(m)} = \frac{D}{\sum_{t=1}^{n} \frac{FCF_t}{(1+WACC_{m-1})^t}}.$$

Then, using equation (9), we find that:

$$w_d^{(m)} = \frac{D}{\sum_{t=1}^{n} \frac{FCF_t}{(1+k_0(1-w_d^{(m-1)}T))^t}}$$

Note that we have obtained a recurrent formula expressing the share of the company's debt as calculated at the mth step of iterations through its same value, but calculated at the previous (m-1)th step of the iterative -process. Thus, we have:

$$w_d^{(m)} = f(w_d^{(m-1)}) \tag{16}$$

Where the function $f(w_d)$ is found from the equation:

$$f(w_d) = \frac{D}{\sum_{t=1}^{n} \frac{FCF_t}{(1+k_0(1-w_d T))^t}} \tag{17}$$

We must prove the convergence of this iterative process, i.e. that $w_d^{(m)} \to w_d^0$ with the growth of the number of iterations *m*. It is obvious that within the limit at $m \to \infty$, from equality (16) we obtain that

$$f(w_d^0) = w_d^0,$$

i.e. the limit value of the company's assets should here be the fixed point of mapping: $y = f(w_d)$. Thus, we must prove that the recurrent process (16) converges to the fixed point of this mapping.

For this purpose, the well-known principle of contracting mappings is suitable (Kolmogorov A.N., Fomin S.V. 1972), according to which, if function $y = f(x)$, which is determined at the interval [a, b], meets the condition of :

$$| f(x_2) - f(x_1) | \le \alpha | x_2 - x_1 |,$$

with constant $\alpha < 1$, and maps the interval [a,b] into itself, then function $y = f(x)$ has a single fixed point x_0, $f(x_0) = x_0$; with any sequence of numbers of the type:

$x_1, x_2 = f(x_1), x_3 = f(x_2), ..., x_m = f(x_{m-1}), ...$ converging to this fixed point, $x_m \to x_0$.

In particular, the condition of contraction is met, if the function has, at interval [a,b], derived function $f'(x)$, with $| f'(x) | \le \alpha < 1$.

Based on the above, we will now investigate the derivative of our function $f(w_d)$.

It is clear that

$$f'(w_d) = -\frac{D}{(\sum_{t=1}^{n} \frac{FCF_t}{(1+k_0(1-w_d T))^t})^2} \cdot \sum_{t=1}^{n} \frac{tFCF_t k_0 T}{(1+k_0(1-w_d T))^{t+1}} =$$
$$= -\frac{Dk_0 T}{V^2(1+k_0(1-w_d T))} \cdot \sum_{t=1}^{n} \frac{tFCF_t}{(1+k_0(1-w_d T))^t}.$$

The, using (9), we have:

$$\begin{aligned} |f'(w_d)| &= \frac{Dk_0 T}{V(1+k_0(1-w_d T))} \cdot \frac{1}{V} \cdot \sum_{t=1}^{n} \frac{tFCF_t}{(1+WACC)^t} = \\ &= \frac{Dk_0 T}{V(1+WACC)} \cdot Dur = \frac{w_d k_0 T}{1+WACC} \cdot Dur, \end{aligned} \tag{18}$$

where

$$Dur = \frac{1}{V} \cdot \sum_{t=1}^{n} \frac{tFCF_t}{(1+WACC)^t},$$

is essentially duration of free cash flows in the project.

But then, inequality $|f'(w_d)| \leq \alpha < 1$ is equivalent to inequality:

$$\frac{w_d k_0 T}{1+WACC} \cdot Dur \leq \alpha < 1, \tag{19}$$

Which should be true at a certain α<1.

This inequality may be presented in the following form:

$$\frac{k_0 - WACC}{1+WACC} \cdot Dur \leq \alpha < 1.$$

The latter inequality is equivalent to inequality

$$\frac{1+WACC}{k_0 - WACC} \geq \frac{Dur}{\alpha},$$

Which can be presented as:

$$\frac{\frac{1+WACC}{WACC}}{\frac{k_0}{WACC} - 1} \geq \frac{Dur}{\alpha}. \tag{20}$$

It should be noted that duration of an arbitrary perpetuitous cash flow (regardless of payments on perpetuity) is calculated from the equation:

$$Dur_{per} = \frac{1+WACC}{WACC}. \tag{21}$$

Thus, the latter inequality is equivalent to the following inequality:

$$\frac{Dur_{per}}{\frac{k_0}{WACC}-1} \geq \frac{Dur}{\alpha}. \tag{22}$$

The inequality (22) constitutes a sufficient condition for convergence of our recurrent process.

Thus, if inequality (22) is true in accordance with the principle of contracting mappings, the iterative procedure converges to the single fixed point of the function $f(w_d), w_d^0$.

And this will mean the existence and uniqueness of the reasonable capital structure for this company/project.

The investigation of realizability of this sufficient condition for the case of a project/company generating only positive cash flow was carried out in the work by (Limitovsky M.A., Minasyan V.B. 2010).

Now let us check the realizability of the sufficient condition for convergence of our iterative process for a project which will generate positive cash flows in future.

Let's consider possible values T=0.20, $k_0 = 0.25$ и $w_d = 0.5$. Then, according to formula (9): $WACC = 0.25 \cdot (1-0.5 \cdot 0.20) = 0.225.$ Using formula (21), we obtain:

$$Dur_{per} = \frac{1+0.225}{0.225} = 5.44 \text{ years}; \ \frac{k_0}{WACC} - 1 = \frac{0.25}{0.225} - 1 = 0.11.$$

If the company generates only positive cash flows, then, as we know, $Dur \leq n$; inequality (22) will be true for such companies if inequality $\frac{5.44}{0.11} \geq \frac{n}{\alpha}$, or $n \leq 49.5 \cdot \alpha$, is true, where α = any figure less than 1 but arbitrarily close to it. This means that if the forecast period n for the project under the specified conditions is equal to approximately 49.5 years or less, then the condition (22) will be met, and the algorithm will provide a single and unique solution.

Now let us carry out an analysis of sensitivity of the approximate life of a project generating positive cash flows depending on parameters k_0 and w_d at typical (for the Russian Federation) tax rate of 20%.

We will obtain the following results.

Table 1 - Analysis of sensitivity of the maximum time of an investment project, during which the convergence of the algorithm is guaranteed, depending on parameters k_0 and w_d

Unlevered rate of return k_0	Share of borrowed capital in the market structure of the capital w_d					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
8%	674	336	224	167	134	111
10%	549	274	182	136	109	90
12%	465	232	154	115	92	76
14%	406	202	134	100	80	66
16%	361	180	119	89	71	59
18%	326	162	108	80	64	53
20%	299	149	99	74	59	49
22%	276	137	91	68	54	45
24%	257	128	85	63	50	42
26%	241	120	79	59	47	39
28%	227	113	75	56	44	37
30%	215	107	71	53	42	35
32%	205	102	67	50	40	33
34%	196	97	64	48	38	31

The table above provides a rough estimate of the limit life (years) of the forecasted period for an investment project or a company generating positive cash flows, during which the algorithm guarantees a uniqueness solution.

The unlevered rate k_0 for companies generating cash flows exceeds 25% extremely seldom; and the share of debt in the market structure of capital of such companies exceeds 50% on equivalently rare occasions.

This means that for a vast majority of actually existing companies generating only positive cash flows the algorithm is characterized by convergence, since in most cases forecasted periods for evaluation of companies generating cash flows do not exceed the estimated limit of 40 years. Note that the convergence is actually proven for all projects and companies, for which cash flow duration does not exceed the maximum time of the project (company).

However, for companies, which generate not only positive cash flows, this statement is not true. Duration for such companies may exceed the maximum time (n) of the project (company). In the work by (Limitovsky M.A., Minasyan V.B. 2010) we did not obtain the sufficient condition for convergence of the iterative algorithm for companies generating not only positive cash flows, because the authors failed to get estimates from a higher level (or find another such valuation in literature) for the duration of a cash flow with other than positive components of such cash flow. To overcome this difficulty, the following assertion was proved in the work:

Assertion:

The following inequality is true for the duration of a cash flow which includes other than positive components:

$$Dur \le n + (n-1)\frac{V^{-}}{V} \quad , \quad (23)$$

Where n = full time of existence of the cash flow; V^{-} = present (zero-moment) sum total of values of modules of negative elements of the cash flow.

Proof

Let's assume that in k periods from n periods of existence of the cash flow, the cash flows are negative, and in the rest (n – k) periods, they are positive. Periods with negative cash flows we will mark as $i_1, i_2, ..., i_k$; whereas periods with positive cash flows, accordingly, we will mark as $i_{k+1}, i_{k+2}, ..., i_n$.

Then, the formula for determining duration will look as follows:

$$Dur = -\frac{1}{V}\cdot\sum_{j=1}^{k}\frac{i_j\,|FCF_{i_j}|}{(1+WACC)^{i_j}} + \frac{1}{V}\sum_{j=k+1}^{n}\frac{i_j FCF_{i_j}}{(1+WACC)^{i_j}} .$$

Now we will introduce the designations:

$$w_{i_j}^{-} = \frac{1}{V}\frac{|FCF_{i_j}|}{(1+WACC)^{i_j}} \quad \text{for j = 1, 2,…, k, and}$$

$$w_{i_j}^{+} = \frac{1}{V}\frac{FCF_{i_j}}{(1+WACC)^{i_j}} \quad \text{for j = 1, 2,…, k.}$$

Then, it is clear that the following equalities are true:

$$-w_{i_1}^{-} - w_{i_2}^{-} - ... - w_{i_k}^{-} + w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+} = 1$$

(24)

$$Dur = -i_1 w_{i_1}^{-} - i_2 w_{i_2}^{-} - ... - i_k w_{i_k}^{-} + i_{k+1} w_{i_{k+1}}^{+} + i_{k+2} w_{i_{k+2}}^{+} + ... + i_n w_{i_n}^{+}$$

(25)

From the equation (25), it obviously follows that:

$$Dur < -w_{i_1}^{-} - w_{i_2}^{-} - ... - w_{i_k}^{-} + n(w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+}).$$

Then, using simple conversions and (25) several times, we will get:

$$Dur < -w_{i_1}^{-} - w_{i_2}^{-} - ... - w_{i_k}^{-} + w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+} + (n-1)(w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+}) =$$

$$= 1 + (n-1)(w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+}) =$$

$$= 1 - w_{i_1}^{-} - w_{i_2}^{-} - ... - w_{i_k}^{-} + (n-1)(w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+}) + w_{i_1}^{-} + w_{i_2}^{-} + ... + w_{i_k}^{-} =$$

$$= 2 + (n-2)(w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+}) + w_{i_1}^{-} + w_{i_2}^{-} + ... + w_{i_k}^{-} =$$

$$= 2 - w_{i_1}^{-} - w_{i_2}^{-} - ... - w_{i_k}^{-} + (n-2)(w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+}) + 2(w_{i_1}^{-} + w_{i_2}^{-} + ... + w_{i_k}^{-}) =$$

$$= 3 + (n-3)(w_{i_{k+1}}^{+} + w_{i_{k+2}}^{+} + ... + w_{i_n}^{+}) + 2(w_{i_1}^{-} + w_{i_2}^{-} + ... + w_{i_k}^{-}) = \ldots =$$

$$= n + (n-1)(w_{i_1}^{-} + w_{i_2}^{-} + ... + w_{i_k}^{-}).$$

Recalling the expressions for $w_{i_j}^{-}$, we will get:

$$Dur < n + (n-1)\frac{1}{V}\sum_{j=1}^{k}\frac{|FCF_{i_j}|}{(1+WACC)^{i_j}} = n + (n-1)\frac{V^{-}}{V}.$$

The assertion is thus proven.

If we use (22) and the proven assertion, it will be clear that trueness of inequality (22) for such companies will be guaranteed if the following inequality is true:

$$\frac{Dur_{per}}{\frac{k_0}{WACC} - 1} \geq \frac{n + (n-1)\frac{V^{-}}{V}}{\alpha}.$$

The latter inequality is equivalent to the following:

$$n \le \frac{1}{1+\frac{V^-}{V}}\left(\alpha\left(\frac{Dur_{per}}{\frac{k_0}{WACC}-1}\right)+\frac{V^-}{V}\right) \tag{26}$$

Inequality (26) constitutes a sufficient condition for convergence of our recurrent process. If we substitute expressions for Dur_{per} and WACC into this inequality, this sufficient condition for convergence (and, accordingly, for existence and uniqueness of the optimum purpose-oriented capital structure) may be re-formulated as follows:

$$n \le \frac{1}{1+\frac{V^-}{V}}\left(\alpha\frac{1+k_0(1-w_dT)}{k_0w_dT}+\frac{V^-}{V}\right).$$

(27)

Now let us check the realizability of the sufficient condition for convergence of our iterative process for companies which will, in future, generate not only positive cash flows.

Let us consider the following values: T=0.20, $k_0 = 0.25$, $w_d = 0.5$. Also, let us assume that $\frac{V^-}{V} = 0.5$. Let us mark, as V^+, the sum of values of positive components of the cash flow as of the present moment (moment zero). The latter equality is equivalent to $\frac{V^-}{-V^- + V^+} = 0.5$, or $V^- = \frac{1}{3}V^+$, which looks rather realistic.

By substituting the selected parameter values into inequality (27), we will obtain:

$$n \le \frac{1}{1+0.5}\left(\alpha\frac{1+0.25(1-0.5\cdot 0.20)}{0.25\cdot 0.5\cdot 0.20}+0.5\right).$$

or $n \le 0.666(49\cdot\alpha + 0.5)$ where α = any figure less than 1 but arbitrarily close to it. This means that if the forecast period n for the project under the specified conditions is equal to approximately 33 years or less, then the condition (27) will be met, and the algorithm will provide a single and unique solution.

Now let us investigate the behavior of the right-hand portion of inequality (27) at all possible realistic values of parameters k_0 , w_d and $\frac{V^-}{V}$ (refer to Tables 2-7).

Table 2 - Analysis of sensitivity of the maximum time of an investment project, during which the convergence of the algorithm is guaranteed, depending on parameters k_0, w_d and $\frac{V^-}{V} = 0.125$ (T = 0.20).

Unlevered rate of return k_0	Share of borrowed capital in the market structure of the capital w_d					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
8%	599	299	199	149	119	99
10%	488	243	162	121	97	80
12%	414	206	137	102	82	68
14%	361	180	119	89	71	59
16%	321	160	106	79	63	52
18%	290	144	96	72	57	47
20%	265	132	88	65	52	43
22%	245	122	81	60	48	40
24%	228	114	75	56	45	37
26%	214	106	71	53	42	35
28%	202	100	66	50	39	33
30%	191	95	63	47	37	31
32%	182	90	60	45	35	29
34%	174	86	57	43	34	28

Table 3 - Analysis of sensitivity of the maximum time of an investment project, during which the convergence of the algorithm is guaranteed, depending on parameters k_0, w_d and $\frac{V^-}{V} = 0{,}25$ (T = 0.20).

Unlevered rate of return k_0	Share of borrowed capital in the market structure of the capital w_d					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
8%	539	269	179	134	107	89

10%	439	219	146	109	87	72
12%	372	186	123	92	74	61
14%	325	162	107	80	64	53
16%	289	144	96	71	57	47
18%	261	130	86	64	51	43
20%	239	119	79	59	47	39
22%	221	110	73	54	43	36
24%	206	102	68	51	40	33
26%	193	96	64	47	38	31
28%	182	90	60	45	35	29
30%	172	86	57	42	34	28
32%	164	81	54	40	32	26
34%	157	78	51	38	30	25

Table 4 - Analysis of sensitivity of the maximum time of an investment project, during which the convergence of the algorithm is guaranteed, depending on parameters k_0, w_d and $\frac{V^-}{V} = 0.5$ (T = 0.20).

Unlevered rate of return k_0	Share of borrowed capital in the market structure of the capital w_d					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
8%	449	224	149	112	89	74
10%	366	183	121	91	73	60
12%	310	155	103	77	61	51
14%	271	135	90	67	53	44
16%	241	120	80	60	48	39
18%	218	108	72	54	43	36
20%	199	99	66	49	39	33
22%	184	92	61	45	36	30
24%	171	85	57	42	34	28
26%	161	80	53	40	31	26

28%	152	75	50	37	30	25
30%	144	71	47	35	28	23
32%	137	68	45	34	27	22
34%	131	65	43	32	25	21

Table 5 - Analysis of sensitivity of the maximum time of an investment project, during which the convergence of the algorithm is guaranteed, depending on parameters k_0, w_d and $\frac{V^-}{V}=1$ (T = 0.20).

Unlevered rate of return k_0	Share of borrowed capital in the market structure of the capital w_d					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
8%	337	168	112	84	67	56
10%	275	137	91	68	55	45
12%	233	116	77	58	46	38
14%	203	101	67	50	40	33
16%	181	90	60	45	36	30
18%	163	81	54	40	32	27
20%	150	75	50	37	30	25
22%	138	69	46	34	27	23
24%	129	64	43	32	25	21
26%	121	60	40	30	24	20
28%	114	57	38	28	22	19
30%	108	54	36	27	21	18
32%	103	51	34	25	20	17
34%	98	49	32	24	19	16

Table 6 - Analysis of sensitivity of the maximum time of an investment project, during which the convergence of the algorithm is guaranteed, depending on parameters k_0, w_d and $\frac{V^-}{V}=2$ (T = 0.20).

Unlevered rate of return k_0	Share of borrowed capital in the market structure of the capital w_d					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
8%	225	112	75	56	45	37
10%	183	92	61	46	37	30
12%	155	78	52	39	31	26
14%	136	68	45	34	27	22
16%	121	60	40	30	24	20
18%	109	54	36	27	22	18
20%	100	50	33	25	20	17
22%	92	46	31	23	18	15
24%	86	43	29	21	17	14
26%	81	40	27	20	16	13
28%	76	38	25	19	15	13
30%	72	36	24	18	14	12
32%	69	34	23	17	14	11
34%	66	33	22	16	13	11

Table 7 - Analysis of sensitivity of the maximum time of an investment project, during which the convergence of the algorithm is guaranteed, depending on parameters в k_0, w_d and $\frac{V^-}{V} = 4$ (T = 0.20).

Unlevered rate of return k_0	Share of borrowed capital in the market structure of the capital w_d					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
8%	135	68	45	34	27	23
10%	110	55	37	28	22	18
12%	93	47	31	23	19	16
14%	82	41	27	20	16	14
16%	73	36	24	18	15	12

18%	66	33	22	16	13	11
20%	60	30	20	15	12	10
22%	56	28	19	14	11	9
24%	52	26	17	13	10	9
26%	49	24	16	12	10	8
28%	46	23	15	12	9	8
30%	43	22	15	11	9	7
32%	41	21	14	10	8	7
34%	40	20	13	10	8	7

The unlevered rate k_0 for companies generating exceeds 25% extremely seldom; and the share of debt in the market structure of capital of such companies exceeds 50% on equivalently rare occasions.

The valuation practice shows that the share of the discounted absolute value of the negative portion of cash flows of such companies in their total discounted cash flows $\frac{V^-}{V}$ seldom goes beyond the limits of 0.125 thru 4. We can see that for projects (businesses) with forecasted periods of 599 thru 7 years, convergence of the iterative process to the single solution is guaranteed. Projects with the minimum guaranteed forecasted period of convergence correspond to the maximum (from those considered) unlevered rate $k_0 = 34\%$, the maximum (from those considered) debt share of 0.6, and the maximum (of those considered) discounted value of negative cash flow in their total discounted cash flows $\frac{V^-}{V} = 4$. Such combination has extremely low probability; but even in such a case, convergence of the iteration process is guaranteed for forecasted periods of up to 7 years.

This means that for a vast majority of really existing companies, convergence of the iteration algorithm is guaranteed by this criterion.

Convergence may be basically proven for other types of projects/companies (e.g. for a company, which is an actual option or an economically separate project, etc). And though this has not been done at this stage of the investigation, our multifarious applications of the suggested algorithms in real projects testify to the fact that they almost always provide the single and unique solution.

Conclusion

The difficulties in calculation of the capital structure, financial leverage, and weighted average capital cost consist in the fact that to evaluate a project or a company one should know the market structure of its capital; whereas, to obtain a reasonable market structure of the capital, valuation results shall be available ((NPV or value of the company). To settle this problem, in article suggest using iterative algorithms for calculating the structure and weighted average capital cost for a company generating cash flows. These algorithms are easily implementable using the popular EXCEL software. The article specifies advantages of such algorithms in comparison with the famous Evans-Bishop and Pratt-Martin iterative techniques. It is proven the suggested algorithms, in the vast majority of real situations, have a single and unique solution for valuation calculations.

The general character of the proven criterion should be noted. In this work, the final numerical test of convergence of the iterative process to a single solution is proven based on notions of the reasonable area of measuring the key parameters which affect the convergence of iterations in the Russian practice (e.g. the chosen tax rate is T = 0.20 etc.). However, using this criterion and notions of the reasonable area of measuring the respective parameters in any other countries, one can also verify, for which companies this algorithm will guarantee convergence to the single solution determining the reasonable structure of capital of each specific company.

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